Giftedness in Mathematics

DEFINITION

It is not helpful to attempt to define precisely the concept of giftedness in mathematics. A pragmatic approach (Haylock, 2004: 151) is to use 'gifted in mathematics' to describe the pupil who turns up from time to time in a primary school year group, for whom an intelligent and thoughtful teacher will recognize that the provision in mathematics that is appropriate for the normal range of pupils in that year group is nothing like sufficiently demanding and challenging. Such pupils will be characterized not just by high attainment in conventional mathematics tests of knowledge, skills, understanding and application, but also by higher-order cognitive abilities, such as analysis, synthesis and creativity in mathematical activity.

EXPLANATION AND DISCUSSION

Mathematics is the core subject in the curriculum where exceptional ability can be most marked. Primary school teachers will always have available additional material for their more able pupils in mathematics, to ensure that they are challenged sufficiently and achieve their potential. But sometimes they come across a pupil for whom their normal stock of additional material seems hopelessly inadequate and insufficiently demanding. Such a pupil may be regarded as gifted in mathematics. But what are the key characteristics of giftedness in this subject?

In his model of multiple intelligences, Gardner (2000) identifies a number of distinct kinds of intelligence, which are relatively autonomous and independent of each other. One of these is 'logical-mathematical intelligence', which includes both analysis (systematic and logical reasoning) and synthesis (recognizing patterns and articulating generalizations). These two key strands of mathematical ability reflect the importance of both analytical and synthetic thinking in mathematics. To solve a problem, the mathematician starts by analysing its components, breaking it up into its various parts, determining what is given and what is the goal. Synthesis involves bringing a number of parts together into a whole, for example, when the mathematician formulates a generalization from a number of specific cases.

These two strands are clearly identifiable in various descriptions of the characteristics of pupils with exceptional ability in mathematics (DfEE, 2000: 4; Kennard, 2001: 100; Krutetskii, 1976: 350–1). To summarize these, the following are the ways in which mathematically gifted pupils are usually distinguished from their peers.

- They grasp new material quickly.
- They use mathematical symbols with confidence and move quickly from concrete to abstract.
- They have an inclination to make connections in mathematics.
- They grasp quickly the formal mathematical structure of a problem.
- They generalize patterns and relationships, often unprompted.
- They generalize an approach to solving one problem and recognize when it can be used in other problems.
- They often leave out intermediate steps when solving familiar problems.
- They are prepared to approach problems from different directions and persist in finding solutions.
- They think flexibly and are not constrained by routines and stereotype procedures.
- They provide logical arguments to explain mathematical results.
- They are able to recall generalized results, principles and methods.

Perhaps surprisingly, such lists of the characteristics of mathematically gifted pupils in primary schools do not include prodigious skill in numerical calculations. Krutetskii (1976) found that this was not a necessary component of high mathematical ability. Exceptional ability in mathematics is not just high achievement in the standard elements of the curriculum. For example, all the various accounts of mathematical ability in school children include reference to the importance of cognitive processes such as flexibility, non-rigidity and non-reliance on stereotype procedures, which are key components of creative thinking in mathematics. Creativity in mathematics is therefore a third strand of giftedness in mathematics, to go with analytic and synthetic thinking. Haylock (2004: 159) argues that mathematically gifted pupils are those who are not just high attaining in conventional terms (skills, knowledge, understanding and application) but also highly creative (flexible, divergent, non-rigid and unconventional) in their approach to mathematical tasks.

Because giftedness in mathematics is characterized by higher levels of thinking such as synthesizing solutions, principles and approaches, generating creative approaches to problem solving, thinking analytically and developing logical arguments, then the special needs of these pupils are not met simply by moving them more quickly through the standard curriculum provided for the majority of pupils. To give scope for the development of the specific characteristics of giftedness in mathematics, the following are guidelines for the kinds of experiences needed by mathematically gifted pupils:

- using their mathematical skills and knowledge to solve unfamiliar problems, including opportunities to articulate or determine the problem;
- engaging with problems with too little or too much information;
- learning to clarify the givens and to clarify the goal, and to identify sub-goals in mathematical problem solving;
- having opportunities to explain their reasoning and justify their solutions;
- learning new mathematical content and skills that will open up opportunities for a broader range of experiences of mathematical thinking and processes;
- exploring number patterns arising from geometrical patterns, and vice versa;
- articulating generalizations in words and with algebraic symbols;
- learning to identify independent and dependent variables and to control independent variables systematically;
- using two-way tables for summarizing and exploring patterns in situations with two independent variables;
- having opportunities to investigate, to conjecture, to hypothesize, and to make higher-level generalization;
- having opportunities to identify general principles and to generalize procedures that arise in one situation that can be applied in others;
- being encouraged to persist in problem solving, to try alternative approaches, to call on a wider range of mathematical concepts and facts;
- engaging in tasks that require logical reasoning;
- engaging in tasks where they need to be systematic;
- engaging in tasks where they succeed by being unorthodox, by not adhering to routines, by being flexible, non-rigid, thinking divergently, and adopting original approaches.

PRACTICAL EXAMPLES

One of the authors of this book undertook some research with a group of able pupils in mathematics that included two pupils who exemplify most clearly the difference between high attainment in conventional mathematics and giftedness in this subject.

J: a high attainer, not gifted

J is judged by the class teacher to be a high attainer in mathematics, has a high level of competence in standard learnt routines and does very well in conventional tests. But the researcher found J to be low in confidence and clearly anxious when presented with unfamiliar problems, hiding work from others and reluctant to take risks. J is relatively poor at recognizing number patterns and making generalizations, is slow in making connections in mathematics, and shows rigidity in thinking. Although J may do well in national assessments, at this stage this pupil is not demonstrating the key indicators of being gifted at mathematics.

K: a gifted underachiever

By contrast, K is a very untidy 9-year-old, with poor handwriting skills. K has not been outstanding in normal mathematics classes. The class teacher regards K as an underachiever, careless, not highly motivated and unwilling to work hard.

In one task, the researcher gave K a sheet taken from a newspaper with pages numbered 35, 36, 109, 110. From this K was able quickly to work out how many pages there were in the newspaper. Given the rest of the newspaper K then investigated how the pages were arranged on the sheets. The researcher equipped the pupil with a simple algebraic idea, using n for the number of pages, m for the number of sheets, f for the number of the front page of the four-page folded sheet, b for the back, l and r for the left and right inside pages. K quickly concluded that $n = m \times 4$. The pupil responded to a suggestion to tabulate the numbers of the pages that came together on each sheet of the newspaper:

f	l	r	b
1	2	71	72
3	4	69	70
5	6	67	68
7	8	65	66

and so on …

K was then able to formulate the following generalizations, such as 'l and b are always even', '$l + b = n + 2$' and '$f + r = n$'. K could then explain how the last of these could be used with any one sheet to find the number of pages. The pupil then looked at other newspapers and was able to explain why these generalizations would work in all cases where there were four pages to each sheet. Summarizing K's achievements on a number of tasks, the researcher concluded that this pupil shows many of the qualities of being gifted in mathematics, in particular:

- exceptional ability in grasping the mathematical structure of a problem and in reasoning mathematically;
- ability to articulate generalizations for number patterns, using all four operations and squares of numbers and algebraic notation;
- an appropriate level of self-confidence, persistence and flexibility in tackling unfamiliar problems;
- a memory for new terms for mathematical concepts and ability to use them correctly in new situations.

FURTHER READING

The starting point for any serious study of the nature of giftedness in mathematics must be Krutetskii (1976). Koshy (2000a) provides both theoretical and practical guidance on teaching mathematics to able children. Porter (2005), although not specific to mathematics, provides a rationale for gifted education, as well as comprehensive and practical guidance for working with able children up to about 8 years of age. Also helpful for this age range is the chapter on supporting the gifted child in mathematics in Edwards (1998). A useful resource for teachers is the internet-based material provided by NRICH (www.nrich.maths.org), a project based at the University of Cambridge School of Education, providing curriculum enrichment and learning support in mathematics for very able children of all ages.